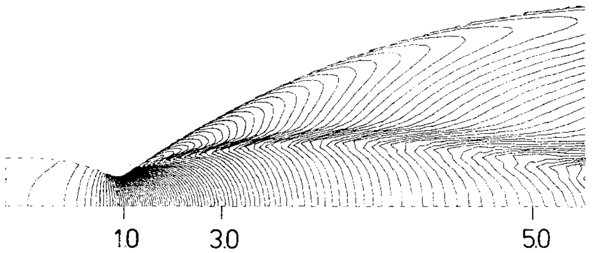

Fig. 3a Mach number ($\gamma = 1.7$).

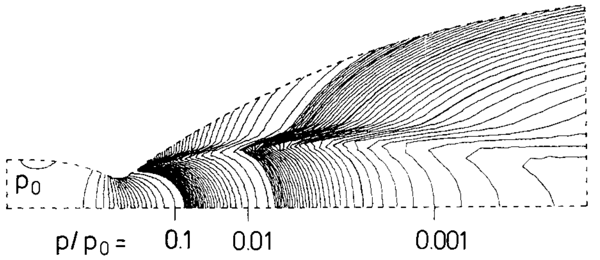

Fig. 3b Static pressure contours ($\gamma = 1.7$).

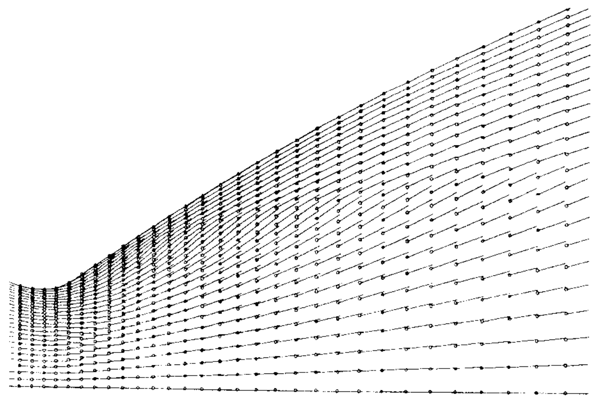

Fig. 3c Velocity vectors near the throat ($\gamma = 1.7$).

and one for $\gamma = 1.17$, using the same geometry. Figure 1 shows the 25×71 grid. The coordinate lines are concentrated in the wall region and near the throat, where large flow gradients are expected. Figures 2a–c show the lines of constant Mach number, the pressure, and the velocity vectors near the throat for the $\gamma = 1.4$ case. A shock wave is observed in the expansion part of the nozzle close to the wall and nearly parallel to it. The maximum exit mach number is about 8.2. The pressure varies about a factor of 500 from the nozzle inlet to exit. This does not cause any difficulties for the applied algorithm during the transient phase. Note that Fig. 2b (see also Fig. 3b) has been obtained by subdividing the nozzle in the axial direction into two (three) domains for the graphics. In each region, a different increment was used for the plot of the isobars to cope with the large axial pressure differences without creating large black or white regions. For the $\gamma = 1.7$ case, Figs. 3a–c show the Mach number, pressure contours, and velocity vectors respectively. Again a shock wave occurs. This shock wave is significantly weaker than for $\gamma = 1.4$, and its location and shape differ considerably from those of the former flow. Here the maximum exit Mach number is about 5.3, and the pressure ratio is roughly 200. The comparison of the two flows shows a large difference in inviscid flow structure due to the different isentropic exponents.

References

[1]Weiland, C., "A Split-Matrix Method for the Integration of the Quasi-Conservative Euler-Equations," Notes on Numerical Fluid-Mechanics, *Proceedings of the Sixth GAMM-Conference on Numerical Methods in Fluid Mechanics*, Vol. 13, Vieweg Verlag, 1986, pp. 383–390.

[2]Riedelbauch, S., "Berechnungen von Düsenströmungen," Diplomarbeit erschienen an der Technischen Universität München, Lehrstuhl für Strömungsmechanik, Berichtsnr. 86/6, 1986.

[3]Whitfield, D. L. and Janus, J. M., "Three-Dimensional Unsteady Euler-Equations Solution Using Flux Vector Splitting," AIAA Paper 84-1551, 1984.

[4]Chakravarthy, S. R., Anderson, D. A., and Salas, M. D., "The Split-Coefficient Matrix Method for Hyperbolic Systems of Gas Dynamic Equations," AIAA Paper 80-0268, 1980.

Rule for Optimizing the Performance of Canted, Scarfed Nozzles

Robert L. Glick*
Talley Defense Systems, Mesa, Arizona
and
Jay S. Lilley†
U.S. Army Missile Command
Redstone Arsenal, Alabama

Nomenclature

A	= area
C_F	= thrust coefficient
F	= thrust
$\dot{m}$	= mass-flow rate
M	= Mach number
p	= pressure
r	= radius
u	= velocity
δ	= cant angle
Γ	= gas dynamic constant
γ	= specific heat ratio
θ	= scarf angle
ρ	= density
ξ	= $1 - \tan\theta\tan\delta$

Subscripts

a	= atmospheric
c	= chamber
e	= exit
opt	= optimum
s	= component normal to nozzle axis
t	= throat
v	= component along missile's axis

Introduction

In several applications of importance to rocket propulsion (tandem motors, thrust reversers, aft guidance links), the propulsive nozzle cannot be colinear with the vehicle's axis, and canted, scarfed nozzles must be employed. There are two basic approaches to predicting the performance of these nozzles: numerical integration of the equations of motion for realistic geometry and flowfield conditions[1,2] or simplification of flowfield and geometry to permit a "closed form" solution.[3] Justification for the latter approach is simplicity and the clear revelation of governing physical principals.

Received Nov. 10, 1986. This paper is declared a work of the U.S. Government and is not subject to copyright protection in the United States.

*Principal Engineer. Associate Fellow AIAA.

†Aerospace Engineer, Propulsion Directorate. Member AIAA.

Figure 1 illustrates the geometry of a canted, scarfed nozzle. For a one-dimensional model, nozzle performance is a function of the uniform, supersonic flow at the entrance to the scarfed extension, scarf and cant angles, and back pressure. Diels[3] has employed this model for a cylindrical extension with the assumption that no exit Mach line is swallowed to show that the criterion for maximum missile axial thrust per unit exit area is

$$M_{e,\text{opt}} = [1+\tan\theta \tan\delta]^{1/2} \tag{1}$$

Although this criterion is correct for the basis stated, it does not result in nozzles that maximize performance, i.e., produce maximum missile axial thrust. The objective of this Note is to present a design rule that maximizes missile axial thrust for the general case of the canted, scarfed nozzle.

Analysis

From Fig. 1, the net thrust acting along the missile's axis is

$$F=\dot{m}u_e + (p_e - p_a)A_e \tag{2}$$

Since the flow at OA is assumed to be uniform and supersonic, the pressure in the scarfed extension OAA' is p_e. Therefore,

$$F_v = F\cos\delta - F_s \sin\delta \tag{3}$$

$$F_s = A_s(p_e - p_a) \tag{4}$$

where A_s is the projection of the scarfed exit area normal to the nozzle axis,

$$A_s = A_e \tan\theta \tag{5}$$

Therefore,

$$F_v/\cos\delta = \dot{m}u_e + (p_e - p_a)\xi A_e \tag{6}$$

where

$$\xi = 1 - \tan\theta \tan\delta \tag{7}$$

If we follow the procedure of Hoffman and Zucrow,[4]

$$\mathrm{d}(F_v/\cos\delta) = \dot{m}\mathrm{d}u_e + \xi(p_e - p_a)\mathrm{d}A_e + \xi A e \mathrm{d}p_e \tag{8}$$

From continuity

$$\dot{m} = (A\rho u)_e \tag{9}$$

and from Bernoulli's equation

$$\mathrm{d}p = -\rho u \mathrm{d}u \tag{10}$$

Consequently, Eq. (8) becomes

$$\mathrm{d}(F_v/\cos\delta)/\mathrm{d}A_e = -A_e(1-\xi)\mathrm{d}p_e/\mathrm{d}A_e + \xi(p_e - p_a) \tag{11}$$

Setting the derivative to zero to identify the relative extremum gives

$$p_e - p_a = (1-\xi)A_e(\mathrm{d}p_e/\mathrm{d}A_e)/\xi \tag{12}$$

However, for isentropic flow

$$\mathrm{d}p/p = \gamma M^2(\mathrm{d}A/A)/(1-M^2) \tag{13}$$

and

$$p_c/p_e = [1+(\gamma-1)M_e^2/2]^{\gamma/(\gamma-1)} \tag{14}$$

Therefore, Eq. (12) can be rewritten as

$$p_e/p_a = \{1+\gamma(1-\xi)\Gamma/[\xi(\Gamma-1)]\}^{-1} \tag{15}$$

When

$$\Gamma = 2/(\gamma-1)\{[(p_c/p_a)/(p_e/p_a)]^{(\gamma-1)/\gamma} - 1\} \tag{16}$$

Equation (15) identifies, implicitly, the pressure ratio that maximizes thrust along the missile's axis, $(p_e/p_a)_{\text{opt}}$.

Results and Discussion

Examination of Eq. (15) shows that when the nozzle is aligned with the axis of the missile, $\delta = 0$, or when the nozzle is not scarfed, $\theta = 0$, the condition for maximum missile axial thrust is that the exit pressure and atmospheric pressures are equal. This result is consistent with the optimization rule for conventional nozzles. In addition, the optimum exit pressure will also be equal to the atmospheric pressure for any case in which the scarf and cant angles are complementary, $\xi = 1$. Consideration of the condition when the scarfed exit plane is parallel to the axis of the missile, $\theta = \pi/2 - \delta$, $\xi = 0$, shows that

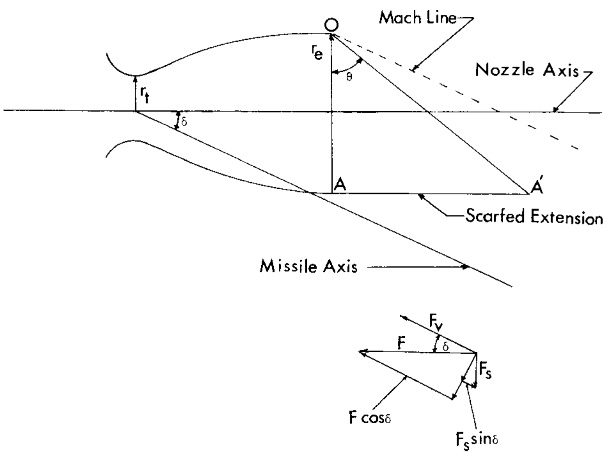

Fig. 1 Scarfed nozzle geometric model.

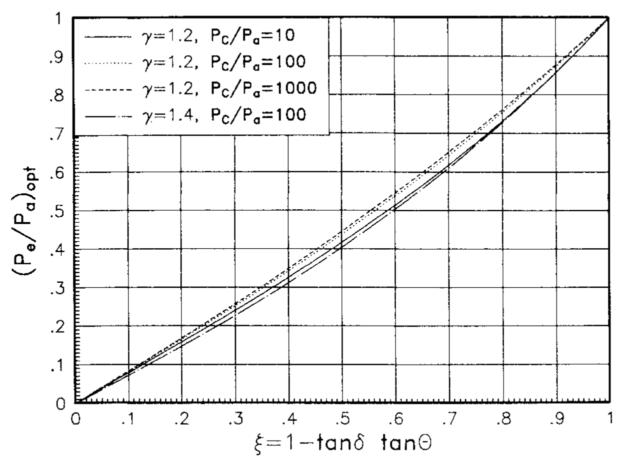

Fig. 2 Optimum pressure ratios.

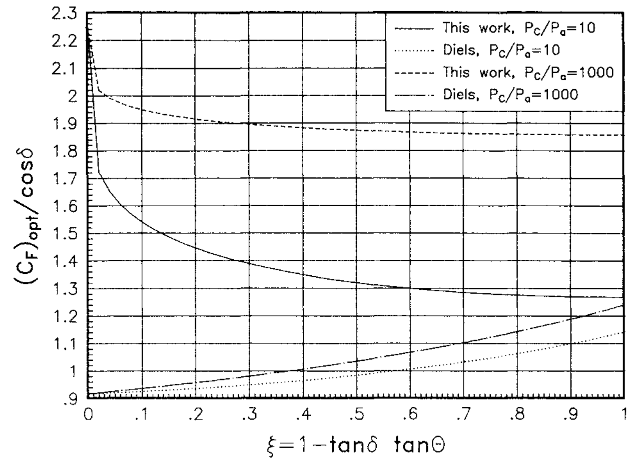

Fig. 3 Optimum performance comparison.

$p_e/p_a = 0$. For this case, there is no optimum (only an asymptotically approached maximum value), and Eq. (6) shows that missile axial thrust is independent of atmospheric pressure. These results agree with the numerical calculations of Lilley and Hoffman.[2] For all other cases, the optimum exit pressure will be less than the atmospheric pressure.

Figure 2 presents results of calculations of the optimal p_e/p_a for a range of chamber pressure and specific heat ratios. The results show weak sensitivity to each of these parameters.

Figure 3 illustrates the variation of a normalized thrust coefficient given by

$$C_F/\cos\delta = F_v/(p_c A_t \cos\delta) \tag{17}$$

as a function of ξ for pressure ratios of 10 and 1000 and $\gamma = 1.2$. The results clearly show that Diels' solution does not maximize the thrust along the missile axis.

Conclusions

The solution for maximum thrust along the missile's axis, found by Diels, maximizes thrust per unit exit area rather than the thrust magnitude. The solution for maximum missile axial thrust has been found for one-dimensional flow in which exit Mach lines are not swallowed. The solution shows that the optimum exit pressure is less than or equal to the atmospheric pressure for a canted, scarfed nozzle. When the exit plane of the canted, scarfed nozzle is parallel to the missile's axis, thrust is independent of atmospheric pressure, and no optimum condition exists.

Acknowledgment

This work was supported by the Ballistic Missile Defense Advanced Technology Center; Program Manager, R. Reviera. Support was also provided by the Propulsion Directorate of the U.S. Army Missile Command.

References

[1]Phillips, W. W., "Thrust Coefficients, Thrust Deflection Angles, and Non-Dimensional Moments for Nozzles with Oblique Exits," Boeing Co., Seattle, WA, Rept. D2-125619-1, March 1968.

[2]Lilley, J. S. and Hoffman, J. D., "Performance Analysis of Scarfed Nozzles," *Journal of Spacecraft and Rockets*, Vol. 23, Jan.-Feb. 1986, pp. 55-62.

[3]Diels, M. F., "Forces on Nozzles with Skewed Exits," Aerojet-General Corp., Sacramento, CA, TM 4510:59-10, Jan. 1958.

[4]Hoffman, J. D. and Zucrow, M. J., *Gas Dynamics*, Vol. 1, Wiley, New York, 1976, p. 235.

Comparing Hydrogen and Hydrocarbon Booster Fuels

James A. Martin*
NASA Langley Research Center
Hampton, Virginia

Introduction

HYDROGEN and hydrocarbon fuels have been compared for use in rocket vehicles in the past. The purpose of this Note is to show more clearly how the differences in the fuels produce differences in vehicles.

Vehicle

The vehicle considered is illustrated in Fig. 1. It could represent an expendable first stage or, with some adjustments, a reusable vehicle. The gross mass m_0 is related to the dry mass m_d by the ideal rocket equation

$$m_0 = m_d e^{v/u}$$

where v is the velocity increment and u the effective exhaust velocity.

The dry mass is the sum of the fixed mass m_f, the tank mass m_t, and the engine mass m_e,

$$m_d = m_f + m_t + m_e$$

The fixed mass represents the payload, upper stages, and other masses that do not change as the propulsion requirements change. The tank mass is a function of the propellant mass $(m_0 - m_d)$

$$m_t = T(m_0 - m_d)$$

where T is a constant for each fuel. The engine mass is a function of the gross mass

$$m_e = E m_0$$

where E is also a constant for each fuel.

These equations can be solved algebraically to give

$$\frac{m_d}{m_f} = \frac{1}{1 + T - (E+T)e^{v/u}}$$

and

$$\frac{m_0}{m_f} = \frac{m_d}{m_f} e^{v/u}$$

Results

The Appendix provides the basis for the selection of the values of the constants given in Table 1. With these constant values, m_0/m_f is shown as a function of velocity in Fig. 2. Figure 3 shows the equivalent results for m_d/m_f.

The gross mass increases with velocity more rapidly with hydrocarbon fuel than with hydrogen fuel. This is expected because of the difference in effective exhaust velocity. While

Received Aug. 7, 1986.

*Aerospace Engineer, Space Systems Division. Associate Fellow AIAA.